LUDWIG-MAXIMILIANS-UNIVERSITÄT MÜNCHEN

MATHEMATISCHES INSTITUT

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Riemannian Geometry

sheet 03

Exercise 1. Show that $(\mathbb{H}, g^{hyp})$ from Ex. 3, sheet 02, has constant sectional curvature -1.

Exercise 2. Recall that a Riemannian metric g is called Einstein with Einstein constant λ if $Ric = \lambda g$. Let (M, g^N) and (N, g^M) be Einstein manifolds with Einstein constants λ_M, λ_N. Show that the product metric on $M \times N$ is Einstein if and only if $\lambda_M = \lambda_N$.

Exercise 3. Let (M, g) be a 4-dimensional Riemannian manifold. Show that g is Einstein if and only if for every point $p \in M$ and every 2-plane $\sigma \subseteq T_pM$ with orthogonal complement $\sigma^\perp$ one has $K(\sigma) = K(\sigma^\perp)$. **Hint:** Consider orthogonal bases for σ and $\sigma^\perp$

Exercise 4. Let (M, g) be a 4-dimensional Riemannian manifold.

1. Assume that for every point $p \in M$ and every 2-plane $\sigma \subseteq T_pM$ with orthogonal complement $\sigma^\perp$ one has $K(\sigma) = -K(\sigma^\perp)$. Show that the scalar curvature of g vanishes identically.
2. Show that the product metric on $S^1 \times S^3$ satisfies the assumption of the previous point.