

Report for the Lehre@LMU program

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In the setting of almost hermitian geometry we may define a class of closed manifolds, called semi-Kähler exact, by imposing that their Kähler form (that is, the two-form determined by the imaginary part of the almost hermitian metric) have an exact power. The exponent of such a power can be readily seen to be one less than half the real dimension of the manifold, in other words, it is one less than the maximal non-zero power. The behaviour of semi-Kähler exact manifolds under riemannian products and conformal transformations was studied. Further, the semi-Kähler exact property was considered in terms of a well known classification of almost hermitian manifolds due to Gray and Hervella. In particular, it is possible to identify two important classes arising in that classification where examples can occur: nearly-Kähler manifolds and balanced manifolds. These two classes have an important role in terms of that classification, as a certain notion of irreducibility is associated to them. Products of Einstein strictly nearly-Kähler manifolds were seen to be semi-Kähler exact, and an explicit primitive was produced. Among balanced manifolds, it was seen that there are examples given by twistor spaces of compact quotients of negative quaternionic-Kähler manifolds, found by taking a one-parameter family of balanced metrics obtained by canonical variation and choosing an appropriate value for the relative scale. Finally, a characterization of semi-Kähler exact manifolds in terms of de Rham currents was found (similar to others already known for Kähler and balanced manifolds) and it was seen how it precludes, in the integrable case, the existence of codimension one complex subvarieties and non-constant holomorphic maps to complex curves.

The help of the program Lehre@LMU allowed the purchase of an important reference used throughout the work.