Ludwig-Maximilians-Universität München
Prof. Dr. Thomas Vogel

SEMINAR ON TOPOLOGICAL K-THEORY
WINTER 2018/19

The main subject of this seminar is the study of vector bundles on compact topological spaces. The classification of vector bundles up to isomorphism is extremely difficult and unknown except in special cases. K-theory can be viewed as an attempt to classify vector bundles up to a coarser equivalence relation. As a result, K-theory associates to each compact topological space a commutative ring.

The main tool for the computation of this ring is the periodicity theorem of Bott. We will discuss Atiyah's proof of this result for complex vector bundles. Using the periodicity theorem one can associate a whole sequence of K-groups to a space with properties very similar to those of singular cohomology. This makes K-theory computable and useful.

Application of K-theory include the classification of real division algebras, and the solution of the question of how many linearly independent vector fields exist on a sphere. Also, K-theory appears in the context of the Atiyah-Singer index theorem. This theorem is not the subject of this seminar, but a description of the relationship between Clifford modules and K-groups is. Clifford algebras are also useful in the construction of linearly independent vector fields on spheres.

- For students of mathematics and physics (Master students and courageous Bachelor students). Although not strictly necessary some knowledge about algebraic topology is an advantage.
- Time and place: Thursday, 12–14 in B133

REFERENCES

[At]   M. F. Atiyah, *K-theory*, W.A. Benjamin 1967.
[Eb]   H.-D. Ebbinghaus et al.,*Zahlen*, 3. Auflage, Springer 1992.
[Ha-K] A. Hatcher *Vector bundles and K-theory*
[LaM]   H. B. Lawson, M.-L. Michelsohn, *Spin geometry*, Princeton Univ. 1989. Press,

TALKS

(1) **Vector bundles, Whitney sum**
- Date: 25.10.
- Speaker: A.F.
- Literature: [At], p. 1-10
- Definition of vector bundles (real and complex), Operations on vector bundles, Whitney sum, Discuss the tautological line, bundle over $\mathbb{CP}^n$.

(2) **Subbundles, quotients**
- Date: 8.11.
- Speaker: A.F.
- Literature: [At], p.10-20
- Subbundles, maps between bundles, pull back, homotopy invariance for paracompact base spaces.

(3) **Vector bundles over compact spaces, clutching functions**
- Date: 8.11.
- Speaker: A.F.
- Literature: [At], p.20-31
- Clutching functions, discuss the clutching function for the tautological bundle over $\mathbb{CP}^1$.

(4) **K-theory, Bott periodicity for complex vector bundles I**
- Date: 15.11.
- Speaker: M.P.
- Literature: [At], p.42–64; [Ha-K], p.38–51.
- Definitions, Formulation of the Bott isomorphism theorem for complex vector bundles, Start of Proof.

(5) **Bott periodicity II**
- Date: 22.11.
- Speaker: P.F.
- Literature: [At], p.42–64; [Ha-K], p.38–51
- End of Proof.

(6) **Cohomology properties of K**
- Date: 29.11.
- Speaker: L.B.
- Literature: [At], p. 66–76;
- K-theory as cohomology theory for compact pairs of spaces, suspensions, exact sequence for pairs.

(7) **Computation of $K(X)$ for some X**
- Date: 5.12.
- Speaker: A.T.
- Literature: [At], p. 76–87 middle.

- Computation of $K(\mathbb{P}(L \otimes \mathbf{1}))$ for complex line bundles L and of $K(\mathbb{CP}^n)$. Discuss the $K(X)$-module structure on $K(X, Y)$

(8) **Classification of real division algebras, parallelizable spheres**
  - Date: 12.12.
  - Speaker: L.G.
  - Literature: [Ha-K], p. 59-66; [Eb], p.234–252.
  - H-spaces, Hopf-invariant of maps $S^{4n-1} \longrightarrow S^{2n}$, real division algebras lead to maps with Hopf invariant ± 1, Adams operations,

(9) **Thom-isomorphism, Splitting principle**
  - Date: 19.12.
  - Speaker: S.V.
  - Literature: [At], 102–116 (omit 103 middle–107 middle).
  - Describe $\tilde{K}(\text{Thom space of } E)$ as $K(X)$-module, splitting principle, Compute $K*(U(n))$, if time permits discuss the Künneth theorem

(10) **Review**
  - Date: 9.1.
  - Speaker: T.V-
  - Literature: [LaM], p. 8–12, 20–25.
  - what has happened so far

(11) **Chern character, J-homomorphism**
  - Date: 16.1.
  - Speaker: A.S.
  - Literature: [Ha-K]
  - Application of K theory to stable homotopy groups of spheres

(12) **K-theory in physics: D-branes**
  - Date: 23.1.
  - Speaker: M.T.