Prof. Dr. Werner Bley Prof. Dr. Cornelius Greither Prof. Dr. Gregor Kemper Prof. Dr. Markus Land Prof. Dr. Andreas Nickel Prof. Dr. Andreas Rosenschon

Sommersemester 2024

## Arithmetische und Algebraische Geometrie

Mittwoch 16-18, LMU Theresienstr. 39, Raum B251 oder TUM, Garching, Boltzmannstr. 3, Raum 02.08.020

17.04.2024 .

Title: Abstract:

24.04.2024 Martin Kreuzer (Passau)

Title: On Border Basis Schemes

Abstract: One of the key features of Algebraic Geometry is the existence of moduli spaces, i.e., of schemes whose closed points correspond to certain types of algebraic varieties or schemes. An intensely studied case is the Hilbert scheme  $\operatorname{Hilb}^{\mu}(\mathbb{P}_{K}^{n})$  parametrizing 0-dimensional subschemes of a fixed projective space over a field K. Performing explicit computer calculations for these schemes has been notoriously difficult, because the presentations of their coordinate rings provided by Grothendieck's construction are hard to make explicit and involve large numbers of indeterminates and defining equations.

Here border basis schemes come to the rescue. For an order ideal  $\mathcal{O}$  of terms, i.e., for a divisor-closed finite set of terms, the border basis scheme  $\mathbb{B}_{\mathcal{O}}$  parametrizes all 0-dimensional affine schemes for which the terms in  $\mathcal{O}$  define a K-vector space basis of their coordinate ring. These schemes form an open covering of the Hilbert scheme and have explicitly describable, well-manageable defining equations.

After recalling the construction and some basic properties of border basis schemes, we survey some recent joint work with Lorenzo Robbiano (Genova) and Le Ngoc Long (Hue) concerning their computational aspects. We consider important subschemes of  $\mathbb{B}_{\mathcal{O}}$  such as the homogeneous border basis scheme, the maxdeg border basis scheme, and various subschemes parametrizing properties such as being locally Gorenstein, strictly Gorenstein, strict complete intersections, having the Cayley-Bacharach property, etc.

The last topic is a new technique, called Z-separating embeddings, for re-embedding schemes from high-dimensional spaces into lower-dimensional spaces which avoids the potentially costly calculation of Gröbner bases and allows us, for instance, to prove that certain border bases schemes are isomorphic to affine spaces and some are not.

08.05.2024 Theodosis Alexandrou (Hannover).

Title: Torsion in Griffiths Groups

Abstract: The Griffiths group  $\operatorname{Griff}^{i}(X)$  of a smooth complex projective variety X is

the group of nullhomologous codimension-i cycles on X modulo algebraic equivalence. Recently Schreieder gave the first examples of smooth complex projective varieties X for which the Griffiths group has infinite torsion. In his examples the infinitely many torsion classes are of order 2. In this talk we show that for any integer  $n \ge 2$ , there is a smooth complex projective 5-fold X whose third Griffiths group contains infinitely many torsion elements of order n.

## 15.05.2024 Robert Pollack (Boston University)

Title: Slopes of modular forms and the ghost conjecture

Abstract: Modular forms are holomorphic functions with a wealth of symmetries. Even though these functions are borne out of complex analysis, their Fourier coefficients contain a wealth of arithmetic information. Even bounding the sizes of these coefficients involve very deep mathematics – the best bounds follow from Deligne's proof of the Weil conjectures, for which he was awarded the Fields medal.

In this talk, rather than looking at complex absolute values, we will instead focus on the p-adic size of p-th Fourier coefficient for a prime number p. We will state a conjecture (the ghost conjecture) which gives an exact description of these sizes for all modular forms. This funnily named conjecture converts difficult automorphic questions into more accessible combinatorial ones. We will discuss the state of this conjecture and its applications to several open questions on slopes of modular forms.

### 22.05.2024 .

Title: Abstract:

### 29.05.2024 .

Title: Abstract:

# 05.06.2024 David Burns (King's College, London)

Title: On the theory of Euler systems

Abstract: We give an overview of work-in-progress with Dominik Bullach concerning the abstract theory of Euler and Kolyvagin systems for p-adic representations. We focus on the motivation for work in this direction and the consequences of our current results for the study of special value conjectures.

### 12.06.2024 .

Title: Abstract:

# 19.06.2024 Lorenz Panny (TUM).

Title: Computing the Deuring correspondence and applications in cryptography Abstract: The Deuring correspondence is a far-reaching equivalence between the theory of supersingular elliptic curves and isogenies, and the theory of maximal orders in a particular quaternion algebra and their ideals. In recent years, isogenies of (usually supersingular) elliptic curves have gained traction in public-key cryptography as a building block for various post-quantum constructions. In this presentation I will explain how to compute the Deuring correspondence efficiently in one direction, and how the presumed hardness of the inverse direction can be used to construct a digital signature scheme known as SQIsign.

#### 26.06.2024 .

Title: Abstract:

### 03.07.2024 Adebisi Agboola (Santa Barbara).

Title: Line bundles on abelian schemes and p-adic heights

Abstract: Let F be a number field with ring of integers  $O_F$ , and suppose that  $E/O_F$  is an abelian scheme. If p is a prime, to what extent is an element of  $Pic^0(E)$  determined by its restriction to p-power torsion subgroup schemes of E? I shall describe an answer to this question that involves a new construction of the p-adic height pairing on E. This is joint work in progress with F. Castella and M. Ciperiani.

## 10.07.2024 Han-Ung Kufner (Regensburg).

Title: Deligne's conjecture on the critical values of L-functions for Hecke characters Abstract: We give a proof of Deligne's conjecture for critical algebraic Hecke characters  $\chi$ , which asserts that the value  $L(\chi, 0)$  agrees, up to a constant in the field of values of  $\chi$ , with a certain period  $c^+(\chi)$ . This generalizes as result of Blasius in the case where  $\chi$  is defined over a CM-field. One of the key inputs in Blasius' proof is an alternative expression of  $c^+(\chi)$  in terms of a period of the so-called reflex motive attached to  $\chi$ .

In our approach, we make use of the recently constructed Eisenstein-Kronecker classes of Kings-Sprang, which allow for a cohomological interpretation of the value  $L(\chi, 0)$ when  $\chi$  is defined over an arbitrary totally imaginary number field. The key insight is that these classes can be naturally regarded as de Rham classes of Blasius' reflex motive. This is achieved via an alternative presentation of the reflex motive. By investigating Blasius' period relation from this new context, the desired relation between  $L(\chi, 0)$  and  $c^+(\chi)$  is established.

17.07.2024 Christoph Winges (LMU).

Title: On the K-theory of categories with bounded t-structures

Abstract: Barwick's theorem of the heart is a devissage-type statement which asserts that the connective algebraic K-theory of a stable infinity-category with a bounded t-structure coincides with the K-theory of its heart. After recalling the relevant notions and explaining Barwick's theorem, I will outline why the analogous assertion for non-connective algebraic K-theory is false in general.

24.07.2024 Eva Brenner und Katharina Novikov (LMU) (two talks a 60min).

16:00 Eva Brenner

Title: On the local epsilon constant conjecture, Galois Gauss sums and normal integral bases

Abstract: In the general framework of the equivariant Tamagawa number conjecture (ETNC), we consider the local epsilon constant conjecture as formulated by Breuning. This conjecture describes the compatibility of ETNC(0) and ETNC(1) with the functional equation of the equivariant Dedekind zeta function. In this talk, we will discuss the role of relations between norm resolvents and Galois Gauss sums in the known proofs. In particular, we will focus on certain wildly and weakly ramified extensions as studied by Bley and Cobbe. The relation that lies at the heart of their proof was established by Pickett and Vinatier. This in turn relies heavily on an explicit construction by Pickett of self-dual normal integral basis generators for the square root of the inverse different, using special values of *p*-adic power series.

# 17:15 Katharina Novikov

Title: On the equivalence of three conjectures from Linear Algebra, Algebraic and Arithmetic Geometry

Abstract: Vaserstein made the conjecture that a particular subgroup of an algebraic K-group of a certain arithmetic ring is torsion. This conjecture can be stated using only the language of Linear Algebra. Bloch has made a similar conjecture in the setting of Algebraic Geometry: For a smooth projective curve over a global field a certain subgroup of an algebraic K-group of the curve is torsion. One goal of this talk is to sketch a proof of the equivalence of these conjectures. To do so we will briefly review the classical definitions of lower K-groups and some exact sequences used in the proof. The second goal is to discuss the connection of the conjecture of Bloch to the conjecture of Bass on finite generation of certain algebraic K-groups. Following results of Kato we show that Blochs conjecture for a curve is a special case of Bass conjecture for a regular proper model of the curve.